When $\varepsilon_0$ is relatively large and the SNR is relatively low, the proposed estimator has two advantages over the CY estimator: much lower computational complexity and superior performance. Thus, under the circumstances where $\varepsilon$ is relatively large and the SNR is relatively low SNR region, the proposed estimator outperforms the CY estimator at rates ranging from 3 to 30 dB. This implies that the larger the $\varepsilon$ is, the wider the range in which the proposed estimator outperforms the CY estimator. Thus, under the circumstances where $\varepsilon$ is relatively large and the SNR is relatively low, the proposed estimator has two advantages over the CY estimator: much lower computational complexity and superior performance.

V. CONCLUSION

We proposed a CFO estimator using ESPRIT in the interleaved OFDMA uplink systems. The CFOs for all active users can be estimated without pilot symbols. When compared to the CY estimator, the proposed estimator outperforms the latter in the relatively low SNR region. In addition, it was shown that the proposed estimator has lower computational complexity. The proposed estimator is therefore more applicable to practical environments than the CY estimator. The simulation results showed very good agreement between the theoretical and the simulated results for the proposed estimator. They also confirmed that the proposed estimator outperforms the CY estimator in the relatively low SNR region.

REFERENCES


Adaptive Robust DOA Estimation for a 60-GHz Antenna-Array System

Moon-Sik Lee, Member, IEEE, Ji-Yong Park, Member, IEEE, Vladimir Katkovnik, Tatsuo Itoh, Life Fellow, IEEE, and Yong-Hoon Kim, Member, IEEE

Abstract—Direction-of-arrival (DOA) estimation in a phased-array system is considered for a non-Gaussian noise environment. In this paper, a robust $M$-estimation scheme based on Huber’s loss function with an adaptive threshold is proposed. The method is evaluated using experimental data from the University of California, Los Angeles, 60-GHz antenna-array system. It is shown that the proposed method enables a decreased sensitivity of the DOA estimates with respect to noise components having heavy-tailed distributions.

Index Terms—Adaptive threshold, non-Gaussian noise, robust direction-of-arrival (DOA) estimation, 60-GHz antenna array

I. INTRODUCTION

An array of antennas has been exploited in numerous applications such as radar, sonar, radio astronomy, and, more recently, wireless communications to improve performance [1]–[3]. Direction-of-arrival estimation is a crucial task in many wireless communication systems.

Manuscript received January 12, 2006; revised July 25, 2006 and January 26, 2007. This work was supported by the University IT Research Center Project, Gwangju Institute of Science and Technology, Gwangju, Korea. The review of this paper was coordinated by Prof. W. Su.

M.-S. Lee is with the Mobile Telecommunication Research Division, Electronics and Telecommunications Research Institute, Daejeon 305-350, Korea (e-mail: moonsiklee@etri.re.kr).

J.-Y. Park and T. Itoh are with the Department of Electrical Engineering, University of California, Los Angeles, CA 90095 USA.

V. Katkovnik is with the Signal Processing Laboratory (TICSP), Tampere University of Technology, Tampere, Finland.

Y.-H. Kim (corresponding author) is with the Department of Mechatronics, Gwangju Institute of Science and Technology, Gwangju 500-712, Korea (e-mail: yhkim@gist.ac.kr).

Digital Object Identifier 10.1109/TVT.2007.900522
(DOA) estimation is one of the important problems in antenna-array processing. Most detection and estimation schemes for radar and wireless communications have been studied under the condition of Gaussian ambient noise, since the Gaussian assumption leads to mathematically tractable solutions [4], [5]. However, in many practical radio environments, the ambient noise is known to be non-Gaussian due to the impulsive nature of man-made electromagnetic interference and a great deal of natural noise [6]–[8]. The non-Gaussian heavily-tailed noise can substantially degrade the performance of conventional systems based on the Gaussian assumption [9], [10].

It has been shown that the minimax $M$-estimation credited to Huber is robust for the unknown noise distribution in the sense that the solution has a decreased sensitivity with respect to variation of the actual noise distribution [11], [12]. It is assumed in the minimax approach that the noise distribution is unknown and belongs to a class of the distributions. Then, the minimax estimator is the best one for the distribution but worst in the class. This minimax $M$-estimation defines a class of estimators which are different from the conventional Gaussian ones in the applied loss functions. While the conventional Gaussian estimators use the “quadratic” loss function, Huber’s robust estimators use special “nonquadratic” loss functions. The estimates obtained by minimization of the sum of nonquadratic functions of residuals have been called $M$-estimates. These new estimators are simple in implementation and well studied theoretically [9], [11], [12]. The efficiency of these sorts of estimators for DOA problems is demonstrated in [13]–[15]. In particular, it is shown in [13] and [14] that the $M$-estimator based on a single-source model appears to have much better resolution than the conventional estimator derived from the quadratic loss function. This multiresolution property is a consequence of using nonquadratic loss function, which is sharper than the conventional quadratic one. Thus, the $M$-estimator demonstrates one more advantage versus the conventional Gaussian estimators.

The utilization of the millimeter-wave frequency bands has received a significant interest for high-speed multimedia wireless communications [16]. In particular, a 60-GHz frequency band has been used in the applications of millimeter-wave wireless local-area network, indoor communications, and intelligent-transport systems such as vehicle-to-vehicle and vehicle-to-roadside communications [16]. Recently, a 60-GHz antenna-array system has been developed for digital-beamforming applications in the University of California, Los Angeles (UCLA) Microwave Electronics Laboratory [16]. The system consists of a 60-GHz four-element linear array with subharmonic inphase/quadrature (I/Q) mixers, an intermediate-frequency (IF) module, analog-to-digital (A/D) converters, and digital-signal processing (DSP).

In this paper, we study a robust $M$-estimator of DOA based on Huber’s nonquadratic loss function. This quadratic-linear loss function is quadratic for small residuals and linear for the residuals having absolute values larger than a threshold. It decreases an influence of large errors in estimation and, in this way, makes the estimates robust with respect to the impulsive noise. The efficiency of this loss function depends on the threshold defining the boundary between the linear and quadratic parts of the loss function. A procedure for the threshold adjustment is proposed. We examine the robustness of the developed adaptive DOA estimator through various experiments using experimental data from the UCLA 60-GHz antenna-array system.

II. ROBUST DOA ESTIMATION

A. Problem Formulation

Consider a uniform linear-phased array composed of $M$ receiving antennas. After the frequency down conversion at the receiver’s outputs, the received-signal model can be expressed in the following conventional vector form [17], [18]:

$$\mathbf{r}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t), \quad t = 1, 2, \ldots, N$$ (1)

where the subindex $k$ indicates the $k$th source, $K$ is the number of sources, $\theta_k$ is the DOA, $s_k(t)$ is the discrete-time source waveform, $N$ is the number of observations, $\mathbf{n}(t)$ is the $M \times 1$ vector array noise, and $\mathbf{a}(\theta_k)$ is the $M \times 1$ steering vector defined as [18], [19]

$$\mathbf{a}(\theta_k) = \left[ 1, e^{-j(2\pi/\lambda) \sin \theta_k}, \ldots, e^{-j(2\pi/\lambda) \sin (M-1) \sin \theta_k} \right]^T$$ (2)

where $\lambda$ is the wavelength, $d$ is the interelement spacing, and $(\cdot)^T$ represents the transpose.

Let us rewrite the multiple-source model (1) in the form commonly used for estimation as

$$\mathbf{r}(t) = \mathbf{a}(\theta_1) s_1(t) + \mathbf{b}(t)$$ (3)

where $s_1(t)$ is a signal from source 1, which is referred to as the desired source, $\mathbf{a}(\theta_1)$ is its corresponding steering vector, and $\mathbf{b}(t)$ denotes “other-signals-plus-noise” vector $\mathbf{b}(t) = \sum_{k=2}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t)$.

B. DOA Estimation

In this section, we develop the robust DOA estimator with an adaptive threshold. We use the following nonquadratic criterion [13]:

$$\psi(s(t), \theta) = \sum_{m=1}^{M} [G(\chi_{I,m}(t)) + G(\chi_{Q,m}(t))]$$ (4)

$$\chi(t) = \mathbf{r}(t) - \mathbf{a}(\theta)s(t)$$ (5)

where $\chi(t)$ is the vector of residuals, $\mathbf{a}(\theta)$ is the steering vector, $s(t)$ is the signal, and the subindexes $I$ and $Q$ stand for the inphase and quadrature components of variables, respectively.

It is known that a special choice of the loss function $G(x)$ in (4) results in the estimates being robust with respect to unknown distributions of the noise components in the observations [9], [11]–[13].

Let us discuss briefly the connections between the $M$-estimates and the minimax robust estimation. Assume that the DOAs $\theta_k$ are independent, random, and uniformly distributed over the interval $[0, 2\pi]$. The covariance matrix $\mathbf{R}$, which is defined as expectation over $\theta_k$ and noise, becomes diagonal as $\mathbf{R} = (K - 1 + \sigma^2) \mathbf{I}$, where $\sigma^2$ is the noise variance, and $E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma^2 \mathbf{I}$. Here, $(\cdot)^H$ represents the Hermitian transpose, and $\mathbf{I}$ is the $M \times M$ identity matrix, which shows that the items of the error-vector $\mathbf{b}(t)$ in (3) are uncorrelated with the variances equal to $(K - 1 + \sigma^2)$. Assume further that the items of the vector $\mathbf{b}(t)$ are independent and identically distributed for all $t$ with some probability density $g_b(x)$. It can be proved that the $M$-estimates of $\theta_1$ and $s_1(t)$, which are derived by minimizing $\psi(s(t), \theta)$ (4), are asymptotically unbiased, and their variances have a common factor

$$V(G, g_b) = \frac{E_b \left\{ \left( G^{(1)}(x) \right)^2 \right\}}{\left( E_b \left\{ G^{(2)}(x) \right\} \right)^2}$$ (6)

where $G^{(1)}(x) = dG(x)/dx$, $G^{(2)}(x) = d^2G(x)/dx^2$, and the expectation $E_b \{ \cdot \}$ is calculated with respect to the density $g_b(x)$.
Thus, the estimation accuracy depends on the loss function $G$ and the noise density $g_b$ only through $V(G, g_b)$. It follows that the analysis and optimization of $V(G, g_b)$ can be used in order to agree a selection of the loss function $G$ with information available on the distribution of the noise.

If the noise density $g_b(x)$ is known, then the minimum of the $V(G, g_b)$ is achieved for $G(x) = G_0(x)$ [11]

$$ G_0(x) = -\log g_b(x). \quad (7) $$

It is the maximum-likelihood (ML) choice of the loss function $G(x)$.

The minimax approach to the problem concerns a much more delicate problem, assuming that the noise density $g_b$ is unknown and, then, the ML solution (7) cannot be used. It is assumed that unknown $g_b$ belongs to the class of densities $\mathcal{P}$, and the optimal loss function is found as a solution of the minimax optimization problem

$$ G_0(x) = \arg \min_G \max_{g_b \in \mathcal{P}} V(G, g_b). \quad (8) $$

Here, the loss function $G_0(x)$ is selected as the best giving the minimal variance, provided that the noise distribution is the worst from the class $\mathcal{P}$. The classes of the robust distributions and the optimal loss functions $G_0$, as well as more details about the minimax robustness concept, can be found in [9], [11], and [12].

We consider the following popular Huber’s quadratic-linear loss function:

$$ G(x) = \begin{cases} \frac{1}{2} \left| \frac{x}{\sigma_0} \right|^2, & \left| \frac{x}{\sigma_0} \right| \leq \mu \\ \mu \left| \frac{x}{\sigma_0} \right| - \frac{\mu^2}{2}, & \left| \frac{x}{\sigma_0} \right| > \mu \end{cases} \quad (9) $$

which is minimax for the class of the approximate Gaussian distributions [11]–[14]

$$ G = \left\{ g_b(x) : g_b(x) = (1 - \alpha) N \left( 0, \sigma_0^2 \right) + \alpha g_1(x) \right\} \quad (10) $$

where $\alpha$ means the probability that the impulse occurs, $0 < \alpha < 1$, and $g_1(x)$ is an arbitrary probability density of this impulse. This loss function is quadratic for small $x/\sigma_0$ and linear for $x/\sigma_0$, having the absolute values larger than the threshold $\mu$. It decreases the influence of large errors in estimation and, in this way, makes the estimates robust with respect to impulse noise. Note that the proposed robust method uses the nonquadratic loss function (9), while the conventional least squares (LS) method uses the quadratic loss function $G(x) = x^2$. The threshold $\mu$ in (9) is related to $\alpha$ and $\sigma_0$ by

$$ \frac{2w(\mu \sigma_0)}{\mu \sigma_0} - 2W(-\mu \sigma_0) = \frac{\alpha}{1 - \alpha} \quad (11) $$

where $W(x) = 1/\sqrt{2\pi} \int_{-\infty}^{x} e^{-t^2/2} dt$, and $w(x) = dW(x)/dx$ [11]. The derivation of (11) is presented in [11]. An adaptive procedure for selection of the threshold $\mu$ is introduced in the next section.

The DOA estimator is derived by minimizing $\psi(s(t), \theta)$; see (4).

The estimate of $s_1(t)$, provided a fixed $\theta$, is a solution of the problem

$$ \hat{s}_1(t, \theta) = \arg \min_{s(t)} \psi(s(t), \theta). \quad (12) $$

In order to solve the problem (12), let us rewrite the criterion (4) in the form

$$ \psi(s(t), \theta) = \sum_{m=1}^{M} \omega_m(t) \left( \chi_{l,m}^2(t) + \chi_{q,m}^2(t) \right) $$

$$ \omega_m(t) = \frac{G(\chi_{l,m}(t)) + G(\chi_{q,m}(t))}{\chi_{l,m}^2(t) + \chi_{q,m}^2(t)} \quad (13) $$

The following recursive reweighted signal estimate minimizing (13):

**Step 0** Initialization by the conventional LS signal estimate [18]

$$ s_1^{(0)}(t, \theta) = \frac{1}{M} A^H(\theta) r(t) $$

$$ W^{(0)}(t) = \text{diag} \{ \omega_1(t), \ldots, \omega_M(t) \} |_{s(t)=s_1^{(0)(t, \theta)}} \quad (14) $$

**Step 1** For $l = 1, 2, \ldots, L$

$$ s_1^{(l)}(t, \theta) = \frac{a^H(\theta) W^{(l-1)}(t) r(t)}{a^H(\theta) W^{(l-1)}(t) \text{diag}(\omega_1(t), \ldots, \omega_M(t))} $$

$$ W^{(l)}(t) = \text{diag} \{ \omega_1(t), \ldots, \omega_M(t) \} |_{s(t)=s_1^{(l)(t, \theta)}} \quad (15) $$

where $L$ is the given maximum number of iterations. The stopping rule of the recursive procedure is $|s_1^{(l)}(t, \theta) - s_1^{(l-1)}(t, \theta)| / |s_1^{(l-1)}(t, \theta)| \leq \zeta$ for some small $\zeta$. Then, the signal estimate is

$$ \hat{s}_1(t, \theta) = s_1^{(l)}(t, \theta). \quad (16) $$

Next, we define the following robust power function:

$$ P_{\text{Rob}}(\theta) = \frac{1}{N} \sum_{t=1}^{N} |s_1(t, \theta)|^2. \quad (17) $$

Finally, we define the estimate of the DOA $\theta_1$ by solving the problem

$$ \hat{\theta}_1 = \arg \max_{\theta} P_{\text{Rob}}(\theta). \quad (18) $$

The optimization in (18) is produced as a search on a grid of values of $\theta$.

In this paper, the performance of the proposed robust DOA-estimation method is compared with the conventional LS method. The signal estimate of the conventional LS method is given by (14).

**C. Adaptive Threshold**

The optimal threshold $\mu$ for the Huber’s loss function (9) is defined by (11). However, the parameters of this equation, which are the variance $\sigma_0^2$ of the Gaussian background noise and the proportion $\alpha$ of the impulse components in the distribution (10), are unknown. We use statistical estimates of these two unknown parameters in order to find the adaptive threshold $\mu$ from (11).

For the estimation of $\sigma_0^2$, we form differences of the observations (1)

$$ \delta_m(t) = r_m(t) - r_{m+1}(t), \quad m = 1, \ldots, M - 1. \quad (19) $$

The preliminary estimate of $\sigma_0$ is given by $\hat{\sigma}_\text{pre} = \text{median}_m ||\text{Re} \{ \delta_m(t) \}|| / \eta$ with $\kappa = 0.675$ obtained as $\kappa = E\{ \text{median}_m(|x(t)|) \} = 0.675$, $x(t) \sim N(0, 1)$. The differences in (19) and the median for estimation of the standard deviation are the standard tools used in order to eliminate or at least
to decrease the influence of the signal on the estimate of the standard deviation of the random noise [13], [14].

The final estimate of $\sigma_0$ is calculated as follows. Replace the residuals $\chi(t)$ in (5) by $\chi(t)/\hat{\sigma}_{pre}$. For a fixed $\mu = \mu_0$, calculate the DOA estimate $\hat{\theta}_1$ (18) and the signal estimate $\hat{x}_1(t) = \hat{s}_1(t, \hat{\theta}_1)$. Then, the new residuals are defined as $\hat{\chi}(t) = r(t) - a(\hat{\theta}_1)\hat{x}_1(t)$, and the final estimate of $\sigma_0$ is obtained as

$$\hat{\sigma}_0 = \text{median}_{t,m} |\text{Re} \{\hat{x}_m(t)\}|, |\text{Im} \{\hat{x}_m(t)\}| \cdot \frac{1}{\kappa},$$

(20)

The parameter $\alpha$ is estimated as a portion of residuals which cannot be explained by the Gaussian distribution

$$\hat{\alpha} = \frac{1}{NM} \sum_{t,m} 1[|\hat{x}_m(t)| \geq 3\hat{\sigma}_0]$$

(21)

where $1[x]$ is an indicator-function; $1[x] = 1$, if $x \geq 0$, and $1[x] = 0$, otherwise. It is assumed in this estimation that the residuals larger than $3\hat{\sigma}_0$ do not belong to the Gaussian distribution $\mathcal{N}(0, \sigma^2)$ in (10).

Finally, we obtain the estimate of the threshold $\mu$ by inserting the estimates, $\hat{\sigma}_0$, which is (20), and $\hat{\alpha}$, which is (21), into Huber’s equation (11)

$$\hat{\mu} = \mu(\hat{\sigma}_0, \hat{\alpha}).$$

(22)

After calculating the threshold $\mu$, the procedure (12)–(18) for the DOA estimation is carried out with this found adaptive value.

III. OBSERVATIONS DESCRIPTION

A. Experimental Data [16]

The 60-GHz antenna-array system that is composed of the four-element integrated linear array, the IF module, A/D converters, and DSP was built at UCLA [16]. Fig. 1 shows the 60-GHz antenna array on alumina substrate including eight output microstrip lines to IF output ports on duroid substrate. The array on alumina substrate within the circle in Fig. 1 is composed of four subharmonic I/Q mixers integrated with each microstrip patch antenna. The use of I/Q channels can significantly reduce the signal-processing load [16]. In order to avoid spatial aliasing problems, an interelement spacing of a half-wavelength might be required. However, it can be difficult to implement a 0.5-λ interelement spacing at 60 GHz due to the limited board space for placing antenna feed line, and so on. The linear array of four antenna elements in the 60-GHz antenna-array system used in the experiments has a 0.6-λ interelement spacing $d = 0.6\lambda$ in (2).

A radio-frequency (RF) signal at 61.875 GHz is down converted to an IF signal at 2.755 GHz by subharmonic I/Q mixers with a first

![Fig. 1. UCLA 60-GHz four-element antenna array. The array is composed of four subharmonic I/Q mixers integrated with each microstrip patch antenna shown in a considerably small circuit size within the circle.](Image)

local-oscillator (LO) signal of 29.56 GHz. In order to obtain down-converted baseband signals of 5 MHz, the IF signals are amplified and mixed with a second LO signal of 2.75 GHz, simultaneously, for $I$ and $Q$ signals at each channel. The baseband signals are then low-pass filtered and amplified. An eight-channel digital oscilloscope is used to sample the baseband $I$ and $Q$ signals by a sampling rate of 200 MS/s.

Experiments were carried out as follows. The antenna-array system receives the signals transmitted through a fixed transmitting antenna, which is located at a distance of 25.4 cm from the receiving antenna to satisfy the far-field condition. In order to receive the source signals from different angles, the receiver array under test is placed on a platform which can rotate from $-90^\circ$ to $90^\circ$. The sampled $I$ and $Q$ signals at each channel are transferred to a personal computer (PC). This experimental data is used for DOA estimation. The signal-processing algorithms are performed in MATLAB in the PC.

The method used in the experiments to calibrate differences between channels is as follows [16]. In order to obtain phase and amplitude information from each antenna element, we convert the sampled data in the time domain to the data in frequency domain by using Hilbert transform. Then, we calculate the differences between channels, and finally, we calibrate circuit errors assuming that each element receives an equal phase and amplitude at broadside.

B. Noise Generation

The experiments were performed in an anechoic chamber where the ambient noise is approximately zero mean Gaussian. In order to evaluate the performance of the proposed method in a non-Gaussian noise environment, we use a computer-generated non-Gaussian noise, which is added to the experimental data from the 60-GHz antenna-array system.

The block diagram of a DOA estimator including the observations description is shown in Fig. 2. In Fig. 2, we use the notation $\tilde{r}_n(t)$ for the experimental data to distinguish them from the observations $r_n(t)$. For the non-Gaussian additive noise, the notation $\tilde{n}_n(t)$ is used. The variables, $\tilde{r}_n(t)$, $\tilde{n}_n(t)$, and $r_n(t)$, are complex-valued.

For the non-Gaussian noise, we use a two-term Gaussian mixture model with the probability density function

$$g_{\alpha}(x) = (1 - \alpha)\mathcal{N}(0, \sigma^2_I) + \alpha\mathcal{N}(0, \sigma^2), \quad 0 \leq \alpha \leq 1$$

(23)
where \( \sigma_1 = \eta \sigma_0 \), and \( \eta \gg 1 \), and \( \mathcal{N}(0, \sigma_0^2) \) and \( \mathcal{N}(0, \sigma_1^2) \) represent the nominal background noise and impulsive noise components, respectively.

The procedure of generating the non-Gaussian noise using the PC is as follows.

1) Set a fixed value of \( \alpha \) which determines a proportion between the nominal background noise and impulsive noise.
2) Set values of standard deviations of \( \sigma_0 \) and \( \sigma_1 \) \( (\sigma_1 = \eta \sigma_0, \eta \gg 1) \), respectively, for the nominal noise and impulsive noise.
3) Generate the \( M \times N \) nominal background noise matrix \( C_0 \) with distribution of \( \mathcal{N}(0, \sigma_0^2) \), where \( M \) is the number of receiving antennas and \( N \) is the number of observations.
4) Generate the \( M \times N \) impulsive noise matrix \( C_1 \) with distribution of \( \mathcal{N}(0, \sigma_1^2) \).
5) Generate an \( M \times N \) indicator matrix \( D \) composed of elements of zero and one for the probability that the impulses occur \( \alpha \), where the number of ones is same as the number of impulses.
6) Finally, generate the non-Gaussian noise using \((O - D) \cdot C_0 + D \cdot C_1\), where \( O \) is the \( M \times N \) matrix of ones.

IV. RESULTS AND DISCUSSION

A. Adaptive Threshold

As mentioned in the previous sections, we consider the linear array of \( M \) = 4 antennas with the 0.6-\( \lambda \) interelement spacing. In all of the experiments, \( N = 50 \).

In Fig. 3, we compare the adaptive threshold (22) versus its accurate optimal value found from (11) for the varying \( \alpha \). We use \( \sigma_0 = 0.1 \), \( \eta = 100 \), and Monte Carlo simulation with the number of runs equal to 50. It is shown in Fig. 3 that the estimates of the threshold \( \mu \) by the proposed algorithm are very close to the corresponding optimal values.

These adaptive-threshold values (22) are used in the subsequent experiments.

B. Beam Patterns

In Fig. 4, we compare beam patterns of the conventional LS and proposed robust methods, respectively, for Gaussian noise \( \alpha = 0, \sigma_0 = 0.1 \) and non-Gaussian noise \( \alpha = 0.1, \sigma_0 = 0.1, \eta = 100 \) in (23). The DOA of the desired source \( \theta_1 \) is 15°. The fine solid line in Fig. 4 is the theoretical beam pattern, which is calculated as

\[ P_{\text{Ideal}}(\theta) = |H^H a(\theta_1)|^2, \]

where \( \mathbf{h} = a(\theta) \) and \( \theta_1 \) are the known DOA, i.e., \( \theta_1 = 15^\circ \).

In the case of Gaussian noise, the two considered methods demonstrate very similar performance. Compared to the ideal pattern, they have slightly higher sidelobe levels. The maximum peaks of their main beams are a bit shifted toward broadside (0°). These performance degradations may be attributed to phase errors and measurement environment.

For the non-Gaussian noise case, the sidelobes of the robust method are still close to those of the ideal pattern while the conventional LS method gives a much higher level of the sidelobes, which confirms that the robust power function \( P_{\text{Rob}}(\theta) \) (17) demonstrates strong resistance to the heavy-tailed noise and preserves the peak’s location close to the true value \( \theta_1 = 15^\circ \).

In Fig. 5, we compare beam patterns of the considered methods in multiple-source environments for non-Gaussian noise \( \alpha = 0.1, \sigma_0 = 0.1, \eta = 100 \). Three sources are considered: \( \theta_1 = -33^\circ, \theta_2 = 0^\circ, \theta_3 = 36^\circ \).

The conventional LS method has the destroyed power response, while the two proposed methods are still close to those of the ideal pattern while the conventional LS method gives a much higher level of the sidelobes, which confirms that the robust power function \( P_{\text{Rob}}(\theta) \) (17) demonstrates strong resistance to the heavy-tailed noise and preserves the peak’s location close to the true value \( \theta_1 = 15^\circ \).

In Fig. 5, we compare beam patterns of the considered methods in multiple-source environments for non-Gaussian noise \( \alpha = 0.1, \sigma_0 = 0.1, \eta = 100 \). Three sources are considered: \( \theta_1 = -33^\circ, \theta_2 = 0^\circ, \theta_3 = 36^\circ \). The conventional LS method has the destroyed power function, which cannot be used for estimation of DOAs of sources, in the non-Gaussian noise case. The proposed robust method continues to give a clear indication of directions from three different sources.
Fig. 6. RMSE performance as a function of $\alpha$ for the conventional LS method and proposed robust method.

Fig. 7. RMSE performance as a function of the DOA of the desired source for the conventional LS method and proposed robust method in non-Gaussian noise with $\alpha=0.1$, $\sigma_0=0.1$, and $\eta=100$.

C. Estimation Accuracy

We use the root-mean-square error (rmse) as the performance criterion to compare the estimation accuracy of the algorithms

$$\text{rmse} = \sqrt{\frac{1}{p} \sum_{i=1}^{p} (\hat{\theta}_1(i) - \theta_1)^2} \quad (24)$$

where $p$ is the number of independent Monte Carlo runs, $\hat{\theta}_1(i)$ is the DOA estimate at the $i$th run, and $\theta_1$ is the true value of the desired DOA.

Fig. 6 shows the rmse performance of the conventional LS method and proposed robust method versus the probability $\alpha$ that the impulse occurs. The rmse of the DOA estimation are computed using 50 independent Monte Carlo simulation runs. We use here $\sigma_0=0.1$, $\eta=100$, and $\theta_1=15^\circ$. It is shown in Fig. 6 that the performance of the robust method remains invariant with respect to the intensity of the level of impulsive non-Gaussian noise. Contrary to the robust method, the conventional LS method is quite sensitive to the impulsive noise and suffers severe performance degradation as $\alpha$ increases.

In Fig. 7, we compare the rmse of the conventional LS method and proposed robust method versus the DOA of the desired source in non-Gaussian noise with $\alpha=0.1$, $\sigma_0=0.1$, and $\eta=100$. The location of the desired source is varied from $-60^\circ$ to $60^\circ$. Initially, the experimental data $\tilde{r}_m(t)$ contains the errors caused by many reasons such as nonideal measurement environment, circuit imbalance, and fabrication error. We confirmed that the estimation errors for the robust method in this impulsive noise are close to those in Gaussian noise. This means that the robust method can robustly estimate the source DOAs in the impulsive noise. The performance degradation near $\pm 60^\circ$ in Fig. 7 is due to the narrow coverage range of the exploited directional antenna elements.

Table I summarizes the rmse results of the two considered methods for different values of $\alpha$, $\sigma_0$, and $\eta$. It confirms a good performance of the robust method. We can see from Table I that the proposed robust method offers significant performance gain over the conventional LS method.

Finally, we can conclude the following.

1) In the case of Gaussian noise, the performance of both the conventional LS method and proposed robust method is almost the same.

2) If $\alpha$ is increasing, i.e., the level of the heavy-tailed random noise is increasing in the observations and the nonrobust conventional LS method degrades quickly, while the proposed robust method continues to give good results.

V. Conclusion

A robust method with an adaptive threshold has been developed for the estimation of the DOAs of source signals in non-Gaussian noise environments. This method has been evaluated using experimental data from the UCLA 60-GHz antenna-array system. It is shown that the proposed robust method can effectively estimate the DOAs in both Gaussian and non-Gaussian noise environments.

### References


